Comment on "Power-law correlations in the southern-oscillation-index fluctuations characterizing El Niño"

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In a recent publication [Phys. Rev. E **63**, 047201 (2001)], Ausloos and Ivanova report power-law probability distributions, fractal properties, and antipersistent long-range correlations in the southern oscillation index. As a comparison with artificial short-range correlated data shows, most of these findings are possibly due to misleading interpretation of the analysis techniques used.

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I. INTRODUCTION

Power laws are found in a wide variety of natural phenomena that feature chaos, self-similarity or criticality [1–3]. However, there is a somewhat disturbing tendency of physicists to find power laws in possibly quite innocuous data, and disregard less exciting alternatives. One recent example of this is a paper by Ausloos and Ivanova [4] in which the authors report power-law correlations in the southern oscillation index (SOI), a climate index connected to the El Niño phenomenon.

I will compare the SOI time series to an artificial dataset obtained from a simple linear autoregressive process using the same methods and representations used in Ref. [4]. It is not my aim to show that all statistical properties of the SOI data can be explained with this model—they cannot. I will, however, show that the methods used to diagnose power laws in Ref. [4] will (falsely) find similar "power laws" in short-range correlated artificial data as well, and are thus not sufficient. The lesson to be learned from this is that statistical properties of a time series should always be compared to surrogate data to make meaningful statements.

II. THE DATA

In Ref. [4], the time series y_t of the monthly averaged SOI from 1866–2000 is studied. I will use the data from Ref. [5], which only has the data from 1866 to 1999, so only $N=1602$ instead of 1612 data points are used. This does not change anything about the conclusions.

The artificial dataset y_t^a for comparison is generated using a first-order linear autoregressive process,

$$y_{t+1}^a = A y_t^a + B\epsilon_t, \tag{1}$$

where ϵ_t are independent Gaussian random variables with mean 0 and variance 1. The parameters A and B are chosen such that the variance $\sigma^2 = \langle (y_t^a - \overline{y_t^a})^2 \rangle$ and the variance of one-step differences $\sigma_d^2 = \langle (y_{t+1}^a - y_t^a)^2 \rangle$ agree with the corresponding values of the SOI data, $\sigma^2 = 1.23$ and $\sigma_d^2 = 0.944$. This fit leads to $A=0.6155$ and $B=0.8732$ (four-digit precision is, of course, unrealistic, but these numbers are as good as any others). The dataset used here is shown in Fig. 1. By construction, it has a well-defined length scale, and its correlation decays as $C(t) \propto \exp(-t/\tau)$, where $\tau = 1/\ln(A) \approx 2.06$ months.

III. DISTRIBUTION OF FLUCTUATIONS

Ausloos and Ivanova first examine the differences $x_t = y_{t+1} - y_t$ by plotting the empirical cumulative probability distribution of $|x|$. They fit a power law, i.e., a line on a log-log scale, to this data in the range $1.5 < |x| < 2.8$ and find a power law with an exponent of $\mu = 3.30$. It should be pointed out that one can fit a line to any sufficiently smooth function, that the fit may even look convincing if the range is small enough, and that most monotonic curves are reasonably smooth on a log-log scale. Thus, in a similar fashion, a line can be fitted to the test data in the same range, suggesting an exponent of $\mu = 4.7$, as seen in Fig. 2.

Of course, the differences x_t^a of the artificial data are Gaussian, and the cumulative probability distribution of their absolute values is $2[1-\Phi(x/\sigma_d)]$. The slope of this function on a log-log plot goes to $-\infty$ as $x \to \infty$—it does not give a power-law value of $\mu = 2$, as Ausloos and Ivanova claim. Thus, one can find tangents of any slope whatsoever by choosing the correct range. For example, the range between

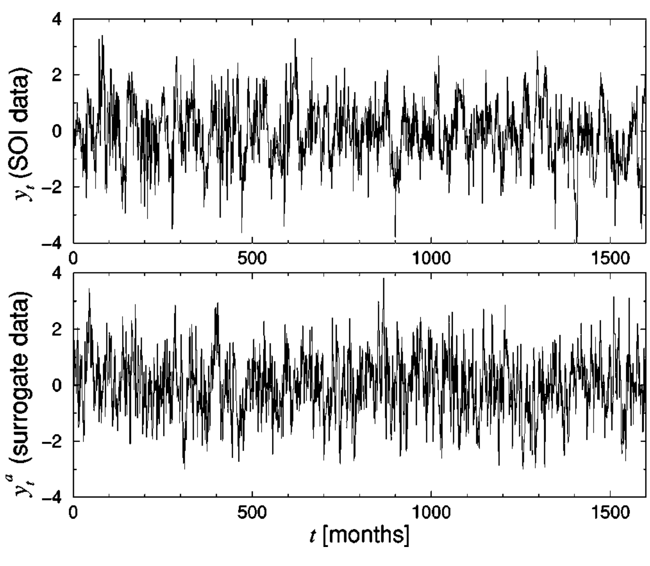

FIG. 1. The SOI dataset (top) and the artificial dataset generated using Eq. (1) (bottom).

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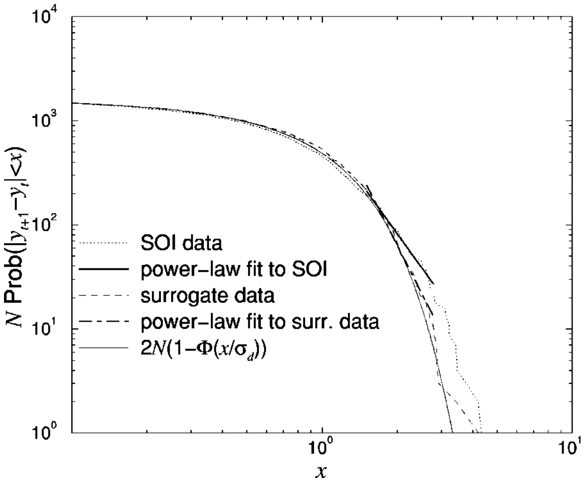

FIG. 2. The cumulative probability distributions $N\mathrm{Prob}(|y_{t+1} - y_t| < x)$ for the SOI data and artificial data are slightly different; however, power law fits over a small range are equally good (or bad) for both. The thin solid line gives the theoretical prediction for the surrogate data.

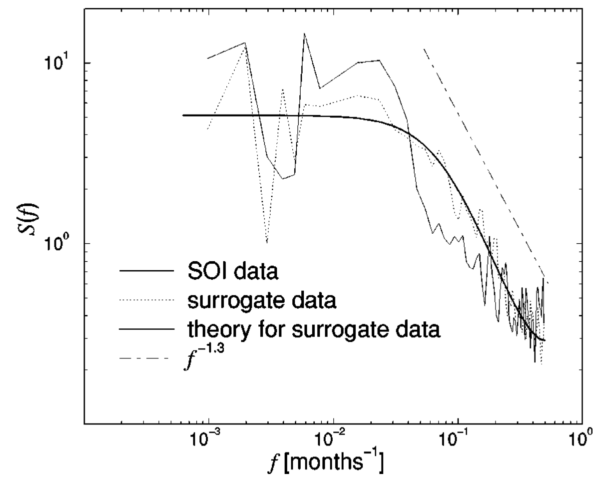

FIG. 3. Estimate of the power spectrum for the SOI data (thin solid line) and the surrogate data (dotted line), and $S(f)$ according to Eq. (2) (thick solid line). The dot-dashed line shows a power law $S \propto f^{-1.3}$ for comparison.

1.1 and 2.2 gives $\mu = 3.3$. Likewise, since the SOI data produce a curved probability distribution, a continuum of slopes can be achieved by choosing the data window for the fit appropriately.

The question whether a given set of data is compatible with a suspected underlying probability distribution can be answered tentatively with statistical tests such as the Kolmogorov-Smirnov (KS) test [6]. In the given case, the KS statistics indicates that the hypothesis that the SOI fluctuations are Gaussian can indeed be rejected on a 1% confidence limit. However, the *ad hoc* hypothesis that the artificial data are drawn from a cumulative distribution that looks like $\Phi(x)$ up to $|x| = 1.5$ and like $Cx^{-4.7}$ for $|x| > 1.5$ cannot be rejected. Neither can any number of other *ad hoc* fits to the empirical distribution—for example, $\mathrm{Prob}(|x_t| < x) = \exp(-x^{1.2}/0.79)$ gives an excellent fit to the SOI data (over the entire range), but without further justification, this is just another meaningless guess.

IV. POWER SPECTRUM

In Ref. [4], the authors then study the power spectrum $S(f)$ of the SOI time series and find a power law $S(f) \propto f^{-\beta}$ with an exponent $\beta = 1.32$ for a frequency range from 1/5 month^{-1} to 1/64 month^{-1}. They draw the conclusion that the SOI data represent a self-affine fractal.

The power spectrum of a function is essentially the modulus square of the Fourier transform of that function, and time series that show self-similarity display power laws in their power spectrum.

However, the statements about smooth functions in log-log plots from the preceding section still hold true here. For example, the autoregressive precdings by which the test data were generated is known [7] to have the power spectrum

$$S(f) = \frac{B^2}{1 + A^2 - 2A\cos(2\pi f/f_{max})}, \qquad (2)$$

where $1/f_{max}$ is the time interval between consecutive data points. Using the parameters given in Sec. I, this function yields a straight line on a log-log plot in the range between 1/2 month^{-1} and 1/20 month^{-1}. One even finds a slope of -1.3 for this line.

Figure 3 shows the function $S(f)$ according to Eq. (2) together with an estimate of the power spectrum of both the SOI data and the test data, calculated with the routines from Ref. [6].

V. DETRENDED FLUCTUATION ANALYSIS

The last analysis in [4] applies the detrended fluctuation analysis (DFA) [8,9], which can be used to determine Hurst exponents in data that contain trends of unknown length scales. The authors report a Hurst exponent of 0.25 ± 0.01 on a scale up to 70 months, at which point the fluctuation function saturates. Their conclusion is that the SOI signal shows antipersistent power-law correlations.

Following Ref. [9], the method of DFA consists of four steps: (1) take the profile (sum) $Y_t = \Sigma_{i=1}^{t} y_i$ of the considered time series y; (2) cut that profile into nonoverlapping segments of length s; (3) fit a polynomial to each of these segments and subtract the fitted function (different orders of polynomials can be used to remove trends of corresponding order—here, order 1 is used exclusively); (4) calculate the variance of the detrended time series within the segments, average the variance over the segments and take the square root. The resulting fluctuation function $F(s)$ shows a power law, $F(s) \propto s^{1-\gamma/2}$, if the underlying time series has long-range correlations of type $C(s) \propto s^{-\gamma}$, and displays $F(s) \propto s^{1/2}$ if the time series is only short-range correlated.

It is interesting to note that the authors of Ref. [4] skip the first step of the procedure, thus doing the DFA of the one-step differences x_t rather than the SOI time series y_t itself. It is hardly surprising that the fluctuation function of x_t saturates at some point: the SOI signal itself takes values within a finite range, no matter for how long one observes it (as one

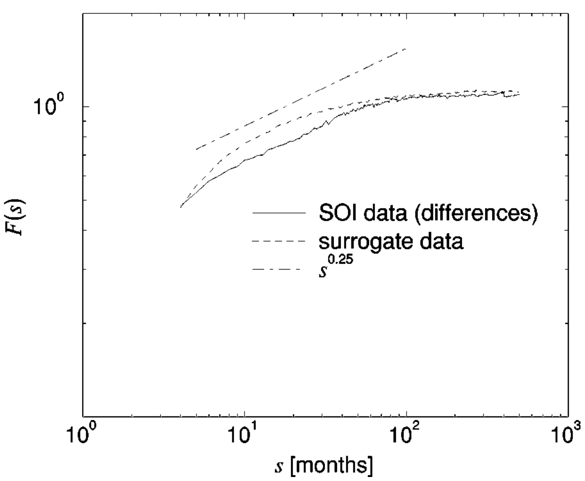

FIG. 4. DFA-1 analysis of the one-step differences $y_{t+1} - y_t$ for both datasets.

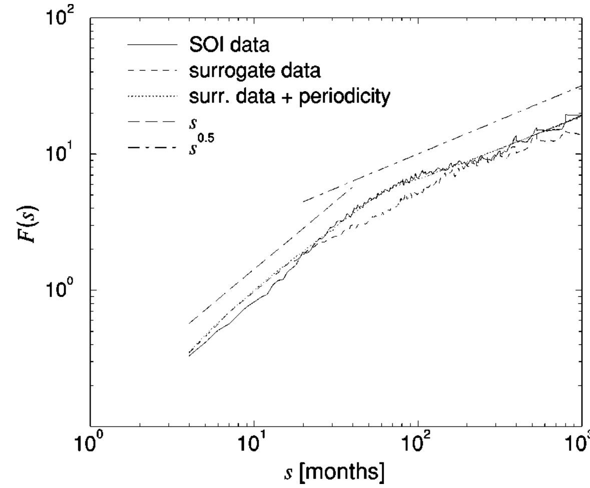

FIG. 5. DFA-1 analysis of the time series y_t for both datasets. Both the SOI data and the surrogate data show the two regimes $F(s) \propto s$ for short times and $F(s) \propto s^{1/2}$ for long times. Including a periodicity into the surrogate data gives a better agreement to SOI data.

should expect for an index that measures air pressure differences), and the sum of the one-step differences over any length of time can never be larger than the difference between the minimal and maximal values of the SOI time series.

Comparing the fluctuation function of the differences of the artificial data to that of the SOI data (see Fig. 4), one notices that there are visible differences between the two curves, and the SOI differences give more of a power-law impression. However, both curves approach their saturation values on similar time scales. It is a slight surprise, but a fact nevertheless, that the $F(S)$ calculated from the (short-range correlated) test data is visibly different from its saturation value up to $s \approx 100$ months.

What about possible long-range correlations in the original time series? A DFA of the time series itself (Fig. 5) shows two regimes for both the SOI and the artificial data. On short times scales, one finds fluctuations compatible with $F(s) \propto s$; random motion $[F(s) \propto s^{1/2}]$ dominates on longer time scales, with a somewhat blurry crossover between them. An estimate of the crossover by fitting power laws to the large-s and small-s regimes and calculating the intersection gives similar crossover times of ≈ 42 months. (It should be noted that DFA systematically overestimates crossover times [9], and that the number of data points is not sufficient to give reliable results in the large-s regime.) There are quantitative differences between the two datasets, especially in the range between 20 and 200 months. These could be explained by a periodicity in the SOI signal with a period of roughly 60 months, a suspicion supported by a hump in the power spectrum at a frequency of roughly 0.02 months^{-1}. To illustrate this, Fig. 5 shows the DFA of a second, longer, set of surrogate data, generated by adding a term $+0.22\sin(2\pi t/60)$ to Eq. (1).

VI. CONCLUSION

The comparison between the SOI data and artificial data with short-range correlations shows that, while the SOI one-step differences display fluctuations whose distribution is not exactly Gaussian, there is little reason to believe that they follow a power-law distribution. Neither is there good evidence that the SOI signal is self-affine. The fluctuation function calculated using DFA shows a regime $\propto s^{1/2}$ for long times, as one would expect for time series without long-range correlations. The shape of the fluctuation function can be partly explained by including a periodicity of roughly 60 months.

More importantly, even with data that are definitely short-range correlated, reasonably straight sections appear on log-log plots of several statistical quantities, which should not be confused with genuine power laws. The willingness to apply the category ‘‘power law’’ to such artifacts can prevent a sufficiently critical analysis of the data and meaningful hypotheses on the underlying mechanisms.

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